Fermi National Accelerator Laboratory

FERMILAB-Conf-98/116

Space-Charge Effects of the Proposed High-Intensity Fermilab Booster

King-Yuen Ng and Zubao Qian

Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

April 1998

Published Proceedings of the *Workshop on Physics at the First Muon Collider and at the Front End of a Muon Collider*, Fermilab, Batavia, Illinois, November 6-9, 1997

Operated by Universities Research Association Inc. under Contract No. DE-AC02-76CH03000 with the United States Department of Energy

Disclaimer

Distribution

Space-charge effects of the proposed high-intensity Fermilab booster

King-Yuen Ng and Zubao Qian

Fermi National Accelerator Laboratory,[1] *P.O. Box 500, Batavia, IL 60510*

Abstract. Space-charge effects on beam stabilities are studied for the proposed two-ring high-intensity Fermilab booster destined for the muon collider. This includes microwave instabilities and rf potential-well distortions. For the first ring, ferrite insertion is suggested to cancel the space-charge distortion of the rf wave form. To control the inductance of the ferrite during ramping and to minimize resistive loss, perpendicular biasing to saturation is proposed.

I INTRODUCTION

The proposed future Fermilab booster has been designed to be used as proton driver for the 50-50 GeV 5×10^{33} cm^{-2}s^{-1} luminosity muon collider also. To accomplish this, the booster should be able to deliver 2 bunches of protons, 5×10^{13} particles each, at 16 GeV with an rms length of $\sim$1 ns and at a repetition rate of 15 Hz [1]. To lower the cost of the rf system, the booster is divided into 2 rings. The lower-energy ring accelerates protons from kinetic energy 1 to 4.5 GeV, has an rf harmonic of 2, a very large aperture, and a small circumference. The higher-energy ring accelerates protons of shorter bunch length up to 16 GeV, has a shorter rf wavelength, and a smaller aperture. A bunch rotation is performed to obtain a shorter bunch of rms length 1 ns just before extraction. Thus two bunch rotations can be performed with the two-ring system if necessary. Some designed parameters [2] for the two rings at injection are listed in Table 1. Laslett tune shifts at injection are given by [3]

$$\Delta\nu = -\frac{3hN_br_p}{2\gamma^2\beta\epsilon_{\rm N95}B} = \begin{cases} -0.393 & \text{1st ring} \\ -0.388 & \text{2nd ring}\,, \end{cases} \tag{1.1}$$

where r_p is the classical proton radius, N_b the number per bunch, h the rf harmonic, $\epsilon_{\rm N95}$ the 95% normalized emittance, and B the *bucket* bunching factor which assumes all buckets were filled. Experience at Fermilab [4], Brookhaven [5], and elsewhere tells us that the above tune depressions are practical when all stop bands are minimized. This has been the criterion with which the two rings are designed and the injection energy of the second ring is chosen [2].

Possible microwave instabilities are discussed in Sec. II. The distortion of the rf wave form by the space-charge force will be computed in Sec. III. To compensate this modification, we suggest in Sec. IV to use an insertion of a hollow ferrite cylinder in the beam pipe. The resistive loss in the ferrite is

---

1) Operated by the Universities Research Association, Inc., under contract with the U.S. Department of Energy.

TABLE 1. Designed parameters for the 2 rings at injection.

	First Ring	Second Ring
Kinetic Energy (GeV)	1.0	4.5
Relativistic gamma γ	2.0658	5.7960
Relativistic beta β	0.8750	0.9850
Cycling rate (Hz)	15	15
Circumference, C (m)	180.649	474.203
Rf harmonic, h	2	21
Number of bunches M	2	2
Number per bunch, N_b	5.0×10^{13}	5.0×10^{13}
Bucket bunching factor, B	0.25	0.25
Transition γ_t	7	25
95% bunch area, A (eV-s)	1.0	1.0
95% normalized emittance, $\epsilon_{\rm N95}$ (π-m)	200×10^{-6}	240×10^{-6}

computed and turns out to be large and position dependent along the bunch unless the bunch is very long. To control the inductance of the insertion and minimize loss, perpendicular biasing to saturation is proposed.

II MICROWAVE INSTABILITIES

The average beam currents at injection are, respectively, $I_{\rm av} = eMN_bf_0 = 23.27$ and 9.977 Amp for the two rings, where $f_0 = \omega_0/(2\pi) = 1.452$ and 0.6227 MHz are the respective revolution frequencies, $M = 2$ is the number of bunches in each ring, and e is the proton charge. With a bucket bunching factor of $B = 0.25$, the peak currents become $I_{\rm pk} = 93.06$ and 419.0 Amp for the two rings. For a parabolic bunch, the line distribution for particles at time τ ahead of the synchronous particle is

$$\lambda(\tau) = \frac{3eN_b}{4\hat\tau}\left(1-\frac{\tau^2}{\hat\tau^2}\right) . \tag{2.1}$$

The half bunch lengths are therefore $\hat\tau = 64.56$ and 14.34 ns for the two rings, or $\hat\ell = 16.94$ and 4.34 m. For a bunch area of 1 eV-s, the half momentum spreads are $\hat\delta = 3.322 \times 10^{-3}$ and 4.208×10^{-3}, respectively. We assume an average betatron amplitude of $\langle\beta\rangle = 25$ m and an average dispersion function of $\langle D\rangle = 1.8$ m. The average beam radius is then

$$a \approx \sqrt{\frac{\epsilon_{\rm N95}\langle\beta\rangle}{\gamma\beta}+\Big(\langle D\rangle\hat\delta\Big)^2} = \begin{cases} 5.29 \text{ cm} & \text{1st ring} \\ 3.33 \text{ cm} & \text{2nd ring}\,. \end{cases} \tag{2.2}$$

It is very probable that a beam pipe or vacuum chamber of radius $b = 8$ and 5 cm will be required, respectively, for the first ring and second ring, because the losses in rings with such high intensities should be minimized to less than 0.1%. The longitudinal space-charge impedances of the beam are then

$$\left.\frac{Z_0^\parallel}{n}\right|_{\rm spch} = i\frac{Z_0}{2\gamma^2\beta}\left(1+2\ln\frac{b}{a}\right) = \begin{cases} 92.1 \text{ Ohms} & \text{1st ring} \\ 10.3 \text{ Ohms} & \text{2nd ring}\,, \end{cases} \tag{2.3}$$

where $Z_0 \approx 377$ Ohms is the free-space impedance. The limits of microwave instability driven by a broad-band impedance are given by the Boussard-modified Keil-Schnell criterion [6]:

$$\left|\frac{Z_0^{\parallel}}{n}\right| < F_{\parallel}\frac{E|\eta|}{e\beta^2 I_{\rm pk}}\left(\frac{\Delta E}{E}\right)^2_{\rm FWHM} = \begin{cases} 75.3 \text{ Ohms} & \text{1st ring} \\ 12.6 \text{ Ohms} & \text{2nd ring}\,, \end{cases} \tag{2.4}$$

where the form factor $F_{\parallel} \approx 1$ for a parabolic bunch, η is the slippage factor, and the full-width-at-half-maximum energy spread is $(\Delta E/E)_{\rm FWHM} = \sqrt{2}(\widehat{\Delta E}/E)$. Different interpretation of these limits can lead to different stability results. Since both the low-energy and high-energy rings operate below transition, any realistic momentum distribution with a slope continuous at the edges of the distribution will enhance the space-charge side of the stability curve. Thus, the Boussard-modified Keil-Schnell limit is usually too pessimistic, and we think longitudinal microwave instability would not happen for the two rings near injection.

From injection to extraction, the slippage factor changes from -0.2139 to -0.0094 for the first ring, and from -0.0282 to -0.0015 for the second. Near extraction, the Keil-Schnell limits reduce to 1.04 and 0.21 Ohms, for the two rings, while the space charge impedances become $Z_0^{\parallel}/n = i16.0$ and $i1.2$ Ohms respectively. Thus flexible momentum-compaction lattices are highly recommended so that the γ_t's can be raised during rampings and lowered back to the design values when bunch rotation or extraction is performed.

A broad-band transverse impedance will also drive the transverse microwave instability. Here, again the most important coupling transverse impedance comes from the space-charge force, which gives the contribution

$$Z_1^{\perp}|_{\rm spch} = i\frac{RZ_0}{\beta^2\gamma^2}\left(\frac{1}{a^2}-\frac{1}{b^2}\right) = \begin{cases} 0.665 \text{ MOhms/m} & \text{1st ring} \\ 0.438 \text{ MOhms/m} & \text{2nd ring}\,, \end{cases} \tag{2.5}$$

where R is the average radius of either ring. A similar Boussard-modified Keil-Schnell limit for transverse microwave instability driven by a broad-band impedance centered at the revolution harmonic n is

$$|Z_1^{\perp}| < F_{\perp}\frac{4\nu\beta}{eRI_{\rm pk}}\left(\Delta E\right)_{\rm FWHM}|(n-\nu)\eta + \nu\xi|\ , \tag{2.6}$$

where the form factor $F_{\perp} \approx 1$ for a parabolic distribution, and ξ is the chromaticity. We use the cutoff harmonics of the beam pipes or vacuum chambers, $n_{\rm cutoff} = 2.405R/b = 987.8$ and 3685, as the central frequencies of the driving impedances for the two rings and $\nu \approx 2.2$ and ~ 12 as the betatron tunes. Then we obtain the limit $|Z_1^{\perp}| < 4.24$ and 4.87 MOhms/m, which are much larger than the respective space-charge values. However, near extraction, the stability limits are close to the space-charge impedances for both rings, due to the smaller slippage factors.

III POTENTIAL-WELL DISTORTION

Knowing the bunch length and the momentum spread, the synchrotron tune can be computed easily,

$$\nu_s = \frac{|\eta|\hat{\delta}}{\omega_0\hat{\tau}} = \begin{cases} 0.001207 & \text{1st ring} \\ 0.002113 & \text{2nd ring}\,. \end{cases} \tag{3.1}$$

Without consideration of the force due to space charge and any other impedance, the required rf voltage required to set up the bucket to fit the bunch is

$$V_{\rm rf} = \frac{2\pi\beta^2 E\nu_s^2}{|\eta|h} = \begin{cases} 31.73\ \text{kV} & \text{1st ring} \\ 250.13\ \text{kV} & \text{2nd ring}\,. \end{cases} \tag{3.2}$$

The rf voltage seen by particle at a time advance τ from the bunch center is (head of bunch is $\tau = +\hat{\tau}$)

$$V_{\rm rf}\sin(-h\omega_0\tau) \approx -V_{\rm rf}\left(\frac{3\pi B}{2}\right)\left(\frac{\tau}{\hat{\tau}}\right) = \begin{cases} -37.38\left(\dfrac{\tau}{\hat{\tau}}\right)\ \text{kV} & \text{1st ring} \\ -\,294.68\left(\dfrac{\tau}{\hat{\tau}}\right)\ \text{kV} & \text{2nd ring}\,, \end{cases} \tag{3.3}$$

where $B = 0.25$ is the bucket bunching factor and the sinusoidal rf has been linearized, the parabolic longitudinal distribution has been assumed. The negative signs in Eq. (3.3) signify that the synchronous phase or stable fixed point is zero for operation below transition.

However, in our situation of low energy and high intensity, the space-charge force shaping the bunch distribution is large and dominates over that due to other coupling impedance. A particle at time advance τ from bunch center sees a longitudinal electric space-charge field

$$E_{z\,\rm spch} = -\frac{eZ_0}{4\pi\beta^2\gamma^2 c}\left(1 + 2\ln\frac{b}{a}\right)\frac{d\lambda}{d\tau}\,, \tag{3.4}$$

where $\lambda(\tau)$ is the line density of the bunch which, for a parabolic distribution, is given by Eq. (2.1). The voltage seen per turn is

$$V_{\rm spch} = E_{s\,\rm spch}C = \frac{3\pi I_b}{(\omega_0\hat{\tau})^2}\left|\frac{Z_0^\parallel}{n}\right|_{\rm spch}\left(\frac{\tau}{\hat{\tau}}\right)\,, \tag{3.5}$$

which gives 29.1 kV at either end of the bunch for the first ring and 14.7 kV for the second ring. Here, $I_b = I_{\rm av}/M$ is the average current per bunch. Comparing Eqs. (3.3) and (3.5), we see that the space-charge distortion of the rf force will be small for the second ring but very large for the first ring. Thus, the rf voltage at injection must be increased to $V_{\rm rf} \approx 56.42$ kV for the first ring in order to cancel the effect of space charge. Another possibility to counteract this space-charge force is to compensate it by using a ferrite insert in the vacuum chamber, which we will study in the next section.

IV FERRITE COMPENSATION

A INDUCTANCE INSERTION

From Eq. (3.5), we see that the effect of the space-charge force on the rf potential can be minimized if the space-charge impedance is canceled by adding an inductance. This idea was first introduced by Neil and Briggs [7], with the wish to mitigate microwave instability, however. If a hollow cylinder of ferrite of length L, inner and outer radii b and d is encircling the beam, an inductive impedance

$$\left.\frac{Z_0^{\|}}{n}\right|_{\rm ind} = -i\frac{Z_0\omega_0}{2\pi c}\mu L\,\ln\frac{d}{b}\,, \tag{4.1}$$

will be introduced, where μ is the relative permeability of the ferrite. For example, with $\mu = 1000$, $b = 8.0$ cm, and $d = 8.8$ cm, a length of $L = 52.96$ cm will be enough to cancel a space-charge impedance of $|Z_0^{\|}/n|_{\rm spch} = 92.1$ Ohms for the first ring at injection.

B LOSSES

Unfortunately, ferrite of high permeability is often accompanied by high resistive losses. A conventional way to introduce loss is to replace the relative magnetic permeability by $\mu \to \mu' - i\mu''$. However, as is shown in Fig. 1, μ'' is highly frequency dependent, being nearly zero at low frequencies and reaching a maximum μ''_R at some high frequency $\omega_R/(2\pi)$. It appears that the simplest representation of the ferrite impedance, which is proportional to $\omega\mu$, may be a broad-band resonance

$$Z_0^{\|}(\omega)\Big|_{\rm ferrite} = \frac{R_s}{1 - iQ\left(\dfrac{\omega}{\omega_r} - \dfrac{\omega_r}{\omega}\right)}\,, \tag{4.2}$$

where Q is the quality factor. The three parameters are to be determined by

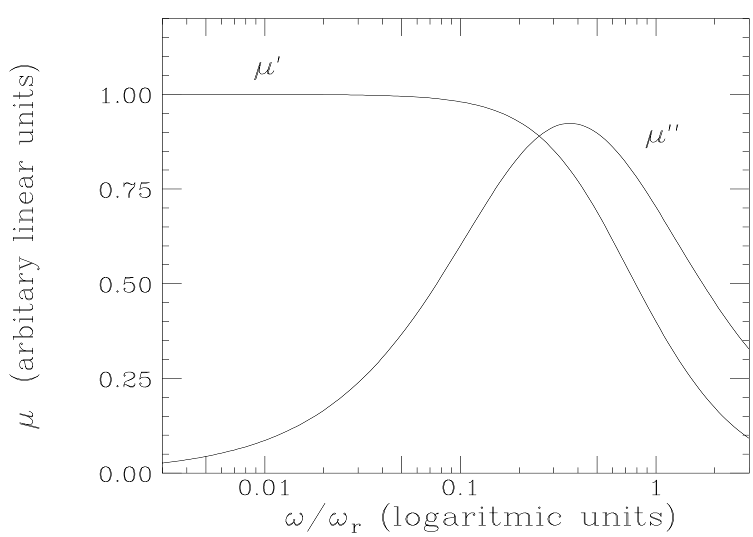

FIGURE 1. A typical plot of μ' and μ'' as functions of frequency.

three measured values of the ferrite, namely μ''_R, ω_R and μ'_L, the latter being the value of μ' near zero frequency. Note that ω_R is the frequency at which Z/n attains the maximum and should therefore be slightly smaller than ω_r. However, we just approximate them to be equal here for simplicity. Equating Eq. (4.2) at low frequencies to the inductive part of the ferrite impedance in Eq. (4.1), the shunt impedance R_s is given by

$$R_s = \frac{Q\omega_r}{\omega_0}\left|\frac{Z_0^\|}{n}\right|_{\rm ind} . \tag{4.3}$$

At resonance, the resistive part gives

$$\mathcal{R}e\; Z_0^\|(\omega)\Big|_{\rm ferrite} = R_s = \frac{\omega_r\mu''_R}{\omega_0\mu'_L}\left|\frac{Z_0^\|}{n}\right|_{\rm ind} . \tag{4.4}$$

Comparing Eqs. (4.3) and (4.4), we obtain $\mu''_R = Q\mu'_L$. Note that Q here relates the values of μ' and μ'' at *different* frequencies, and is not the usual industrial quoted Q which relates them at the *same* frequency.

We are now in a position to compute the loss. For a bunch distribution $\lambda(\tau)$, the energy *gained* per turn for a particle at time advance τ is

$$\Delta\mathcal{E}(\tau) = -e\int_\tau^\infty d\tau'\,\lambda(\tau')W_0'(\tau-\tau')\,, \tag{4.5}$$

where $W_0'(\tau)$ is the longitudinal wake function, which is the Fourier transform of the ferrite impedance of Eq. (4.2). For $\tau<0$, it is given by

$$W_0'(\tau) = \frac{\omega_r R_s}{Q}\,e^{\alpha\tau}\left(\cos\bar\omega\tau + \frac{\alpha}{\bar\omega}\sin\bar\omega\tau\right)\,, \tag{4.6}$$

where the shifted resonant frequency is $\bar\omega = \sqrt{\omega_r^2-\alpha^2}$. Assuming $\omega_r/(2\pi)\approx$ 50 MHz, and $\mu''_R\approx\mu'_L$ (or $Q\approx 1$), the e-folding length of the wake is $\alpha^{-1} = 2Q/\omega_r = 6.4$ ns which is very much shorter than the length of the first-ring bunch at injection. We can therefore expand $\lambda(\tau')$ as a Taylor series about $\tau'=\tau$. Then, except for the very head of a bunch, the energy gained by a particle at τ becomes

$$\Delta\mathcal{E}(\tau) = e\sum_{n=0}\lambda^{(n)}(\tau)\frac{R_s}{Q\bar\omega\omega_r^{n-1}}\sin n\theta\,, \tag{4.7}$$

where $\lambda^{(n)}(\tau)$ is the n-th derivative of λ with respective to τ, $\sin\theta=\bar\omega/\omega_r$ and $\cos\theta=\alpha/\omega_r$. The approximation has been the upper limit of the integration of Eq. (4.5), which should be $\hat\tau$ for a finite bunch instead of ∞. For the parabolic distribution, there are only two terms. For particles that are not too close to the head of the bunch, the loss per turn is

$$\Delta\mathcal{E}(\tau) = \frac{e}{\omega_0}\left|\frac{Z_0^\|}{n}\right|_{\rm ind}\left[\lambda'(\tau)+\frac{1}{Q\omega_r}\lambda''(\tau)\right]\,, \tag{4.8}$$

where the Eq. (4.3) has been used. Substituting Eq. (2.1), we obtain

$$\Delta\mathcal{E}(\tau) = -\frac{3e\pi I_b}{(\omega_0\hat{\tau})^2}\left|\frac{Z_0^\parallel}{n}\right|_{\rm ind}\left(\frac{\tau}{\hat{\tau}}\right) - \frac{3e\pi I_b}{Q\omega_r\omega_0^2\hat{\tau}^3}\left|\frac{Z_0^\parallel}{n}\right|_{\rm ind} . \tag{4.9}$$

The first term is the contribution of the inductance of the ferrite, which cancels the space-charge voltage of Eq. (2.3) if $|Z_0^\parallel/n|_{\rm ind}$ is chosen to equal to $|Z_0^\parallel/n|_{\rm spch} = 92.1$ Ohms for the first ring. The second term gives the average loss of energy per particle per turn. When the space-charge force is canceled, this amount to 1.43 keV. The power loss is

$$P = \frac{3\pi I_b^2}{Q\omega_r\omega_0^2\hat{\tau}^3}\left|\frac{Z_0^\parallel}{n}\right|_{\rm ind} , \tag{4.10}$$

or 16.7 kw per bunch. The energy loss per particle is small at injection. We note that the loss is inversely proportional to the third power of the bunch length. As the protons are ramped to higher energies, the bunches becomes shorter. For example, when the bunches are prepared for extraction into the second ring, the half bunch length may become 14.34 ns or 4.5 times shorter. However, the longitudinal space-charge impedance will be 5.8 times smaller. Thus the energy loss per particle per turn will be increased by roughly 14 times to 20.2 keV. Usually $\mu_R'' < \mu_L'$ and ω_R may be smaller. Thus, the energy loss per particle can be tremendous. Also for a more accurate integration of Eq. (4.5) in the case of a short bunch, the amount of energy loss will depend on the position along the bunch, making compensation impossible. For this reason, it will be best to reduce the ferrite loss to a minimum, which we shall study next.

C PERPENDICULAR BIAS AT SATURATION

When the beam is ramped from the kinetic energy of 1 GeV at injection in the first ring to the kinetic energy of 4.5 GeV for extraction, the space charge impedance will be reduced by a factor of 8.862. We would like the inductance of the ferrite insertion to decrease by the same factor during the ramp. This can be accomplished by passing a dc bias field through the ferrite. To reduce loss, we suggest that the bias field should be perpendicular to the ac magnetic field generated by the beam particles. This dc biased field can be easily provided by placing a solenoid outside the ferrite cylinder.

One way is to set the dc biased field H_c in the beam- or z-direction so high that the magnetization $\vec{M}$ inside the ferrite is saturated and becomes M_s in the same direction. The ac field $\vec{H}_1$ from beam particles is in the transverse or x-y plane. This will produce an ac magnetization $\vec{M}_1$ which precesses about $\hat{z}H_c$ at the gyromagnetic circular frequency of $\omega_c = \gamma_g H_c$ where $\gamma_g = 2\pi \times 2.80$ MHz/Oersted. This precession creates an ac magnetization $\vec{M}_1$ in the transverse plane. Since the ferrite is at saturation, there will not be any hysteresis loss. Thus, we have

$$\vec{H} = \hat{z}H_c + \vec{H}_1 , \qquad \vec{M} = \hat{z}M_s + \vec{M}_1 . \tag{4.11}$$

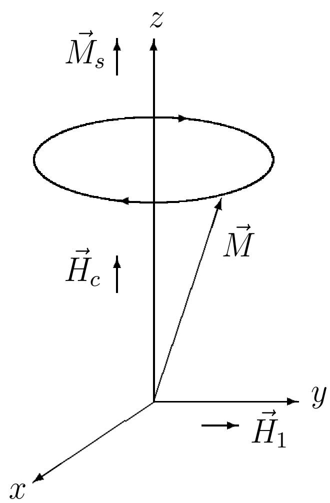

FIGURE 2. System with saturated perpendicular bias H_c in the z-direction. With the application of the ac field $\vec{H}_1$ in the y-direction, the magnetization $\vec{M}$ acquires an ac component in the x-y plane precessing about the z-axis.

The dc biased field is very much larger than the ac field from the beam, or $|\vec{H}_1| \ll H_c$. The system is represented schematically in Fig. 2. The equation of motion is then approximately

$$\frac{d\vec{M}}{dt} = \gamma_g(\hat{z}M_s\times\vec{H}_1 + \vec{M}\times\hat{z}H_c)\,. \tag{4.12}$$

Defining the reversible magnetic susceptibility tensor $\overleftrightarrow{\chi}_r$ as $\vec{M}_1 = \overleftrightarrow{\chi}_r\vec{H}_1$, the stationary solution of Eq. (4.12) is

$$\overleftrightarrow{\chi}_r = \begin{pmatrix} \chi & -j\kappa & 0 \\ j\kappa & \chi & 0 \\ 0 & 0 & 1 \end{pmatrix}\,, \tag{4.13}$$

where

$$\frac{\chi}{\mu_0} = \frac{\omega_c\omega_m}{\omega_c^2-\omega^2}\,, \qquad \frac{\kappa}{\mu_0} = \frac{\omega\omega_m}{\omega_c^2-\omega^2}\,, \qquad \omega_c = \gamma_g H_c\,, \qquad \omega_m = \gamma_g\frac{M_s}{\mu_0}\,, \tag{4.14}$$

with μ_0 being the magnetic permeability of free space. There is a resonance at the gyromagnetic resonant frequency $\omega_c = \gamma_g H_c$, which is proportional to the dc H_c. This explains why we want H_c to be large so that the resonance effect can be avoided. Then, we obtain from Eqs. (4.14), $\kappa/\chi \approx \omega/\omega_c \ll 1$, implying that the off diagonal elements in $\overleftrightarrow{\chi}_r$ can be neglected.

The merit of this saturated biasing is the low loss, because the ferrite is saturated, there will not be hysteresis loss. The only loss is due to spin-wave propagation which is small. The disadvantage is that μ' is usually small at or above saturation. The loss is usually introduced by the factor α through $\omega_c \longrightarrow \omega_c - i\omega\alpha$ and write $\chi = \chi' - i\chi''$. In our application here, the ac field comes from the beam particles. So ω has the range of the bunch spectrum. For example, with the rms bunch length of 28.9 ns, the bunch frequency $\omega/(2\pi)$

has an rms value of ~ 5.5 MHz. It is more convenient to write χ' and χ'' as functions of ω instead with H_c held constant:

$$\chi' - i\chi'' = \frac{\left(\frac{\omega_m}{\omega_c}\right)\left[1-(1-\alpha^2)\left(\frac{\omega}{\omega_c}\right)^2\right] + i\alpha\left(\frac{\omega_m}{\omega_c}\right)\left(\frac{\omega}{\omega_c}\right)\left[1+(1+\alpha^2)\left(\frac{\omega}{\omega_c}\right)^2\right]}{\left[1-(1+\alpha^2)\left(\frac{\omega}{\omega_c}\right)^2\right]^2 + 4\alpha^2\left(\frac{\omega}{\omega_c}\right)^2}.$$

As an example, we choose Ferramic Q-1, which has a saturated flux density $B_s = 3300$ Gauss at $H_c = 25$ Oersted. Thus, the saturated magnetization is $M_s = 3275$ Gauss. At injection, we bias at $H_c = 25$ Oersted, which gives a resonant frequency of $\omega_c/(2\pi) = 70$ MHz, which is very much larger than the bunch spectrum spread. We see from Fig. 3 that up to 15 MHz, $\mu' \sim M_s/H_c = 131$. With the ferrite cylinder of inner and outer radii 8 and 10 cm, a length of $L = 2.96$ m is required to cancel $|Z_0^{\parallel}/n|_{\rm spch} = 92.1$ Ohms. At extraction, μ' must be reduced to $131/8.862 = 14.78$. The biased field should therefore be raised to $H_c = M_s/\mu' = 221.6$ Oersted.

At low frequencies, the loss is $\mu'' \longrightarrow \alpha\omega\omega_m/\omega_c^2$. Taking a typical value of $\alpha = 0.05$, we find μ'' varies linearly from 0 and reaches 1.5 at 15 MHz when $H_c = 25$ Oersted at injection. This is illustrated in Fig. 3. At extraction, the loss is reduced by a factor of $8.862^2 = 74.30$ when $H_c = 221.6$ Oersted. Note that $\chi' - i\chi''$, when multiplied by the proper factor to become an impedance, is in the form of the resonant impedance given by Eq. (4.2) with $Q \approx 1/(2\alpha) \approx 10$ and $\omega_r/(2\pi) \approx \omega_c(2\pi) \approx 70$ MHz. Therefore, the loss per particle per turn at injection or extraction, according to Eq. (4.9), will be about 14 times less than the corresponding values quoted in Sec. IV.B.

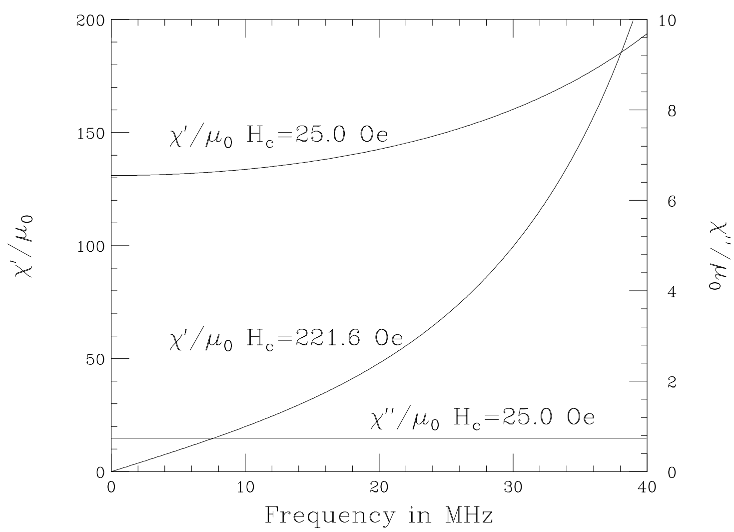

FIGURE 3. The real part of the ac susceptibility of the ferrite as functions of the beam frequency for the dc bias field 25.0 Oersted at injection and 221.6 Oersted at extraction. The imaginary part is shown only at 25.0 Oersted because it is too small to show at extraction.

V CONCLUSION

We have studied the single-bunch instabilities of the two rings of the proposed high-intensity Fermilab booster, and found that the bunches are stable against longitudinal and transverse microwave instabilities near injection, but can be unstable near extraction. The flexible momentum-compaction lattices are recommended, so that the γ_t's can be tuned larger during ramping to assure stability, and are brought down again only when the bunches are rotated at extraction.

For the first ring, the space-charge force will modify the rf waveform by very much, and a ferrite insertion is suggested so that the inductance can compensate the space-charge force. In order to control the ferrite induction during ramping and minimize resistive loss, perpendicular biasing at or above saturation is proposed. In this way, the gyromagnetic resonance arriving from large bias field will have a high frequency well above the frequency spread of the particle beam. Also the hysteresis loss in the ferrite can be avoided.

It is possible that the bunch areas in both rings will turn out to be 1.5 to 2 eV-s instead of 1 eV-s. However, the bunch lengths must be kept to the designed values in order that a 1 ns bunch can be delivered. In other words, the space-charge force will not be changed. If the bunch area is S times larger, the momentum spreads will be S times larger, which will certainly help in combating microwave instabilities. The rf voltages required without consideration of space charge will be S^2 times larger. This imply that the space-charge distortions of the rf potentials will be less severe. Nevertheless, the energy or power lost to the ferrite will remain unchanged.

REFERENCES

1. The $\mu^+\mu^-$ Collider Collaboration, *$\mu^+\mu^-$ Collider: A Feasibility Study*, June 18, 1996, BNL-52503, Fermilab-Conf.-96/092, LBNL-38946.
2. C. Ankenbrandt, private communication.
3. J. Laslett, Proc. 1963 Summer Study on Storage Rings, BNL-7534, p.324, 1963.
4. C. Ankenbrandt and S. Holmes, Proc. 1987 IEEE Part. Accel. Conf., Washington, DC, 1987, p.1066.
5. W.C. Weng, private communication.
6. E. Keil and W. Schnell, CERN Report TH-RF/69-48 (1969); V.K. Neil and A.M. Sessler, Rev. Sci. Instr. **36**, 429 (1965); D. Boussard, CERN Report Lab II/RF/Int./75-2 (1975).
7. V.K. Neil and R.J. Briggs, Plasma Physics **9**, 631 (1966).